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Published in:
Journal of Physics: Conference Series (Online)

Link to article, DOI:
[10.1088/1742-6596/625/1/012030](https://doi.org/10.1088/1742-6596/625/1/012030)

Publication date:
2015

Document Version
Publisher's PDF, also known as Version of record

[Link back to DTU Orbit](#)

Citation (APA):
Murcia Leon, J. P., Réthoré, P-E., Natarajan, A., & Sørensen, J. D. (2015). How Many Model Evaluations Are Required To Predict The AEP Of A Wind Power Plant? *Journal of Physics: Conference Series (Online)*, 625, [012030]. <https://doi.org/10.1088/1742-6596/625/1/012030>

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2015 J. Phys.: Conf. Ser. 625 012030

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How Many Model Evaluations Are Required To Predict The AEP Of A Wind Power Plant?

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Abstract. Wind farm flow models have advanced considerably with the use of large eddy simulations (LES) and Reynolds averaged Navier-Stokes (RANS) computations. The main limitation of these techniques is their high computational time requirements; which makes their use for wind farm annual energy production (AEP) predictions expensive. The objective of the present paper is to minimize the number of model evaluations required to capture the wind power plant's AEP using stationary wind farm flow models. Polynomial chaos techniques are proposed based on arbitrary Weibull distributed wind speed and Von Misses distributed wind direction. The correlation between wind direction and wind speed are captured by defining Weibull-parameters as functions of wind direction. In order to evaluate the accuracy of these methods the expectation and variance of the wind farm power distributions are compared against the traditional binning method with trapezoidal and Simpson's integration rules.

The wind farm flow model used in this study is the semi-empirical wake model developed by Larsen [1]. Three test cases are studied: a single turbine, a simple and a real offshore wind power plant. A reduced number of model evaluations for a general wind power plant is proposed based on the convergence of the present method for each case.

1 Introduction

The evaluation of the performance of a wind power plant requires to calculate the expected energy production over the years. The most common measure of wind farm performance used in the planning or evaluation stages is the annual energy production. AEP is proportional to the expected mean power over all the possible atmospheric conditions: $AEP = 8760 \mathbb{E}(P)$ in [W.h]. The expected power is defined in eq. 1.1. The likelihood of occurrence of the different atmospheric conditions is represented by the joint probability density function, $\mathbb{PDF}(u, \theta)$, where u and θ are the Reynolds-averaged wind speed and wind direction.

$$\mathbb{E}(P) = \int_0^{2\pi} \int_0^{\infty} P(u, \theta) \mathbb{PDF}(u, \theta) du d\theta \quad (1.1)$$

The common practice in the wind energy industry is to consider the Reynolds-averaged wind speed variations over the years at a given location to follow a Weibull distribution. On the other hand, the mean wind direction is modeled by defining sectors or bins. Additionally, the correlation between wind direction and wind speed is captured by defining different wind



speed Weibull-parameters for each sector. As a result the wind speed and wind direction are independent in each of the R_θ wind direction sectors: $[\theta_j, \theta_{j+1}]$, see eq. 1.2. In this equation, \mathbb{P}_j is the the probability of occurrence of each sector.

$$\text{PDF}(u, \theta) \approx \text{PDF}(u|\theta \in [\theta_j, \theta_{j+1}]) \text{PDF}(\theta|\theta \in [\theta_j, \theta_{j+1}]) \mathbb{P}_j \quad (1.2)$$

This assumptions implies that the mean power can be computed in each sector independently and then weighted averaged with respect \mathbb{P}_j .

$$\mathbb{E}(P) \approx \sum_{j=1}^{R_\theta} \mathbb{P}_j \left(\int_{\theta_j}^{\theta_{j+1}} \int_0^\infty P(u, \theta) \text{PDF}(u|\theta \in [\theta_j, \theta_{j+1}]) \text{PDF}(\theta|\theta \in [\theta_j, \theta_{j+1}]) du d\theta \right) \quad (1.3)$$

There are different approaches to numerically approximate eq. 1.3: trapezoidal and Simpson's integration rules consist in building linear or quadratic interpolation lines between a number of wind speed and direction evaluation points, $P(u_k, \theta_k)$. Such techniques are the common practice in the industry and they can be interpreted as a weighted average of the evaluation points, see eq. 1.4. The weights shown in this equation, w_k , depend on the integration technique and on the $\text{PDF}(u, \theta)$.

$$\mathbb{E}(P) \approx \sum_{j=1}^{R_\theta} \mathbb{P}_j \left(\sum_{k=1}^N w_k P(u_k, \theta_k) \right) \quad (1.4)$$

Polynomial chaos with semi-spectral collocation techniques can be used to have a higher accuracy prediction to integrals in the same way as presented in eq. 1.4 with a reduced number of model evaluations. The idea originally introduced in [2], presents quadrature rules based on a normal distribution by the construction of an orthonormal polynomial basis with respect the normal probability density function. The generalized polynomial chaos (gPC) techniques presented in [3] expand this technique to uniform, beta and gamma probability distributions. In the present work, data driven polynomial chaos (aPC) techniques introduced in [4] are applied to the Weibull distribution. Furthermore, the multi-element polynomial chaos presented in [5] is implemented in this paper to deal with integrals over multiple regions of wind speed and wind direction.

2 Polynomial Chaos for AEP

2.1 Multi-element Data-driven Polynomial Chaos (MEPC)

PC techniques enables the user to find the statistical properties of a model with a reduce number of simulations. This is achieved by defining a set of polynomials that are used to interpolate the model response. The purpose of using multiple element PC is to separate the integration region into small sub-regions in which the integration variables can be assumed to be independent. In the present section the theory for building quadrature rules is presented. The theory is presented as a tool to integrate the function $f(x)$ under a random variable, x , characterized by its probability density function $\text{PDF}(x)$. The integral is separated over R_x regions, defined by its end points, x_i .

$$\int_{-\infty}^{\infty} f(x) \text{PDF}(x) dx = \sum_{i=1}^{R_x} \mathbb{P}_i \int_{x_i}^{x_{i+1}} f(x) \text{PDF}(x|x \in [x_i, x_{i+1}]) dx \quad (2.1)$$

An inner product is defined based on the conditional probability density function such that:

$$\mathbb{E}_i(f) = \langle f \rangle_i = \int_{x_i}^{x_{i+1}} f(x) \mathbb{PDF}(x|x \in [x_i, x_{i+1}]) dx \quad (2.2)$$

$$\mathbb{E}_i(fg) = \langle f, g \rangle_i = \int_{x_i}^{x_{i+1}} f(x) g(x) \mathbb{PDF}(x|x \in [x_i, x_{i+1}]) dx \quad (2.3)$$

A polynomial of order n is defined as:

$$\pi_n(x) = a_{0n} + a_{1n}x + \dots + a_{nn}x^n = \sum_{m=0}^n a_{mn}x^m \quad (2.4)$$

An orthonormal polynomial basis based on the probability distribution consists in a group of polynomials that are orthogonal and that have unitary norm with respect the inner product defined in eq. 2.3. These conditions are summarized in eq. 2.5-2.6. These equations are recursive as they use the coefficients from the lower order polynomials to find the next one. The first polynomial is assumed to be: $\pi_0(x) = 1$. Furthermore the statistical moments of the truncated distribution, $\mathbb{E}_i(x^m)$, up to the $(2j-1)$ -th order are required.

$$\langle \pi_n, \pi_k \rangle_i = 0 = \sum_{m=0}^n a_{mn} \langle x^m, \pi_k \rangle_i = \sum_{m=0}^n a_{mn} \left(\sum_{l=0}^k a_{lk} \mathbb{E}_i(x^{m+l}) \right) \quad \forall k < n \quad (2.5)$$

To close this system of equations orthogonality is solved first. This is done by setting the last coefficient of each polynomial to be: $a_{nn} = 1$. Finally the polynomials are normalized:

$$\pi_n = \frac{\pi_n}{\sqrt{\langle \pi_n, \pi_n \rangle}} \quad (2.6)$$

Note that the polynomial basis will be different for each region, since each one has a different conditional PDF. The polynomial basis for the i -th region will be denoted as π_{in} .

The fundamental theorem of Gaussian quadrature states that integrals with respect the PDF(x) can be approximated using only N evaluation points, x_k . The quadrature rules calculate exactly the integral of functions, $f(x)$, that are a polynomial of order equal or smaller than $2N-1$.

$$\mathbb{E}_i(f) = \langle f \rangle_i = \int_{x_i}^{x_{i+1}} f(x) \mathbb{PDF}(x|x \in [x_i, x_{i+1}]) dx \approx \sum_{k=1}^N w_k f(x_k) \quad (2.7)$$

where the evaluation points, x_k , are the roots of the N -th polynomial in the basis, π_{iN} , and the weights, w_k , are computed as:

$$w_k = \frac{1}{\sum_{n=0}^N (\pi_{in}(x_k))^2} \quad (2.8)$$

By projecting any function, f , into the orthonormal polynomial basis one can approximate its response as a polynomial. Note that since the method can handle different truncation orders in each region in general there are going to be discontinuities in the polynomial response

between the regions; this does not produce any problem as the integration in each region is done independently.

$$f(x) = \sum_{n=0}^{\infty} \langle f, \pi_{in} \rangle_i \pi_{in}(x) \approx \sum_{n=0}^N \langle f, \pi_{in} \rangle_i \pi_{in}(x) = \sum_{n=0}^N c_{in} \pi_{in}(x) \quad \forall x \in [x_i, x_{i+1}] \quad (2.9)$$

The coefficients of the function in the polynomial basis, c_{ij} , can be computed numerically using the Gaussian quadrature rules presented in eq. 2.8; this method is also known as spectral collocation:

$$c_{in} = \langle f, \pi_{in} \rangle_i \approx \sum_{k=1}^N w_k f(x_k) \pi_{in}(x_k) \quad \forall n < N \quad (2.10)$$

Finally the statistical moments of the function can be obtained from its coefficients in the polynomial basis:

$$\mathbb{E}(f) = \sum_{i=1}^{R_x} \mathbb{P}_i \mathbb{E}_i(f) = \sum_{i=1}^{R_x} \mathbb{P}_i c_{i0} \quad (2.11)$$

$$\mathbb{V}(f) = \sum_{i=1}^{R_x} \sum_{k=0}^N \mathbb{P}_i c_{ik}^2 - \mathbb{E}(f)^2 \quad (2.12)$$

It is very important to notice that the polynomial surrogate function is not used for computing neither the $\mathbb{E}(f)$ nor $\mathbb{V}(f)$. In fact, the $\mathbb{E}(f)$ can be computed using the quadrature rule directly as presented in eq. 2.7; which is equivalent to computing only the 0-th order coefficients; and then using eq. 2.11. If the variance is required then all the coefficients in the polynomial basis are computed and used in eq. 2.12.

2.2 MEPC for wind speed and wind direction:

Due to the independence between wind direction and speed inside a direction sector, a polynomial basis can be constructed independently for wind speed and for wind direction. The 2D polynomial basis is then the product between the one dimensional polynomial basis. Similarly the 1D quadrature weights can be used to define the 2D quadrature rules for each sector. A polynomial basis of orders N_u for wind speed and N_θ for wind direction are built by solving the equations 2.5 and 2.6. To build the orthonormal polynomials the statistical moments of the truncated Weibull are required, as well as the truncated moments for the wind direction distribution. Expressions for the truncated statistical moments of the Weibull distribution are presented in [6].

$$\mathbb{E}(P) \approx \sum_{i=1}^{R_u} \sum_{j=1}^{R_\theta} \langle P \rangle_{ij} \mathbb{P}_j \mathbb{P}_i \quad (2.13)$$

$$\begin{aligned} \langle P \rangle_{ij} &= \int_{\theta_j}^{\theta_{j+1}} \int_{u_i}^{u_{i+1}} P(u, \theta) \text{PDF}(\theta | \theta \in [\theta_j, \theta_{j+1}]) \text{PDF}(u | u \in [u_i, u_{i+1}]) du d\theta \\ &\approx \sum_{k=1}^{N_u} \sum_{l=1}^{N_\theta} w_{jl} \nu_{ik} P(u_k, \theta_l) \end{aligned} \quad (2.14)$$

The notation used is: w_{jl} represents the weight associated to the l -th root of the N_θ -th order polynomial for the wind direction inside the j -th sector. ν_{ik} represents the weight associated to the k -th root of the N_u -th order polynomial for the wind speed inside the i -th sector. The roots and weights are computed independently as in the single variable case from eq. 2.8. Once the polynomial families have been obtained for each sector, one can capture the statistical properties of the power as produced by the random wind speed and wind direction by projecting the power curve function into the 2D polynomial basis:

$$P(u, \theta) \approx \sum_{k=0}^{N_u} \sum_{l=0}^{N_\theta} c_{ijkl} \pi_{ik}(u) \pi_{jl}(\theta) \quad \forall \theta \in [\theta_j, \theta_{j+1}] \quad \text{and} \quad \forall u \in [u_i, u_{i+1}] \quad (2.15)$$

The coefficients of the power in the polynomial basis can be computed numerically using the quadrature rules:

$$c_{ijkl} = \langle P, \pi_{ik} \pi_{jl} \rangle_{ij} \approx \sum_{k=1}^{N_u} \sum_{l=1}^{N_\theta} w_{jl} \nu_{ik} P(u_k, \theta_l) \pi_{ik}(u_k) \pi_{jl}(\theta_l) \quad (2.16)$$

Finally the statistical moments of the power distribution can be obtained from the polynomial basis coefficients:

$$\mathbb{E}(P) = \sum_{i=1}^{R_u} \sum_{j=1}^{R_\theta} \mathbb{P}_i \mathbb{P}_j (c_{ij00}) \quad (2.17)$$

$$\mathbb{V}(P) = \sum_{i=1}^{R_u} \sum_{j=1}^{R_\theta} \left(\mathbb{P}_i \mathbb{P}_j \sum_{k=0}^{N_u} \sum_{l=0}^{N_\theta} c_{ijkl}^2 \right) - \mathbb{E}(P)^2 \quad (2.18)$$

3 Single wind turbine case

A single wind turbine case is considered in order to verify the concept of using MEPC techniques to estimate the AEP. For this case the power is only a function of the Weibull distributed wind speed. The wind speed operational range of the wind turbine and optionally the region with rated power are used to define the regions of integration. Some examples of the orthonormal polynomial basis for the truncated Weibull distribution are shown in figure 1, as well as the obtained quadrature points and the polynomial surrogates. In this figure it can be observed that the polynomial response passes through the evaluation points. And it can be expected that oscillations might appear for high polynomial orders, especially for regions with constant power.

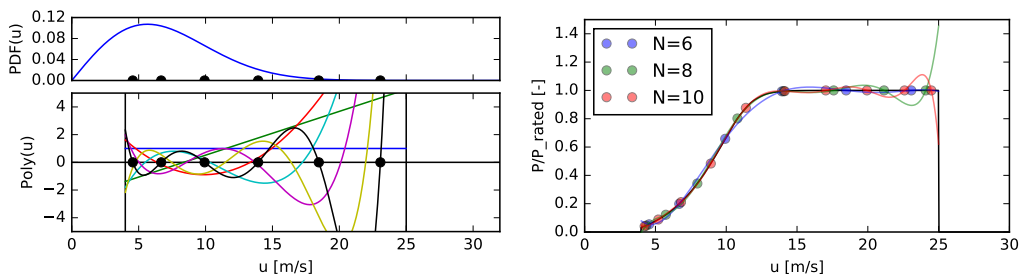


Figure 1. (Left) 6-th order polynomial basis for an individual region Weibull distributed wind speed: $u \in [4, 25]$ [m/s] (Right) Power curve (black line), quadrature evaluation points and polynomial surrogate of the power curve.

Figure 6 shows the convergence for different integration schemes for the relative error of $\mathbb{E}(P)$ and of $\mathbb{V}(P)$ vs. the number of model evaluations used in the integration N . The relative error is calculated with respect the trapezoidal integration rule with 10^5 points. The instabilities in the MEPC quadrature results are produced by the difference in the location of the evaluation points. The location of the evaluation points and the accuracy of the quadrature rules depend on the Weibull parameters because the method depends on the $\mathbb{PDF}(u)$. The benefit obtained for using MEPC is considerable; as it reduced the number of model evaluations from 21 (current common practice) down to 6 model evaluations for similar relative errors of 0.1% in expected power and variance. Note that the method becomes numerically unstable at high polynomial orders (larger than 10) even with careful handling of number precision because of the large differences in the values of the high order statistical moments, $\mathbb{E}(x^m)$. The results presented in this article use the following python libraries: the polynomial handling class on numpy (<http://www.numpy.org>), scipy (<http://www.scipy.org>) for its implementation of the trapezoidal/Simpson's methods and mpmath (<http://mpmath.org/doc/current/index.html>) for handling the polynomial construction and root finding algorithms with an arbitrary high precision.

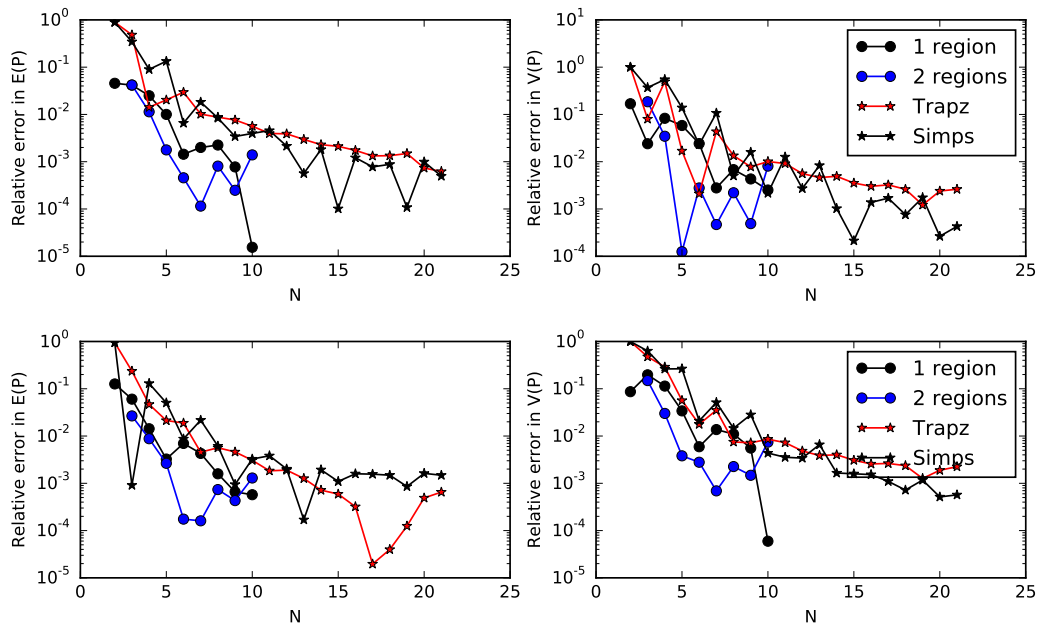


Figure 2. Convergence for different integration schemes vs. number of model evaluations N for the relative error of (left) $\mathbb{E}(P)$ and (right) $\mathbb{V}(P)$. Weibull distribution with shape parameter $k_W = 2$ and scale parameter (Top) $A = 8.0$ [m/s] (Bottom) $A = 10.0$ [m/s].

4 Simple wind power plant case

In this section the multi-element polynomial chaos technique is applied to a simple wind power plant. The power is computed using G. C. Larsen's semi-empirical wake model [1]. The local weather is characterized by a Weibull distributed wind speed and an uniformly distributed wind direction inside each wind direction sector. Note that this type of assumption is the standard in most wind energy flow models such as WAsP. On the other hand, the method proposed in this paper could be applied for an arbitrary distribution of wind direction inside each sector. The only requirement is that the statistical moments are known or computable (some examples of

marginal distributions for wind direction are: Multiple mixture of Von Mises, or kernel density estimated PDF).

Under the assumptions of the present model, the correlation between wind speed and wind direction is modeled by having different parameters of the Weibull distribution as a function of the wind direction sector. The 1D polynomial basis for the wind direction are the Legendre polynomials since the conditional probability density function for the wind direction inside each sector is assumed to be uniform. The final number of evaluation points is the tensorial product (meshgrid combination) of the wind speed and the wind direction quadrature points.

The layout of the simple offshore power plant is shown in figure 3, as well as the power and thrust coefficient curves for the Vestas V80-offshore turbines. The wind rose and Weibull parameters for the site are shown in figure 4.

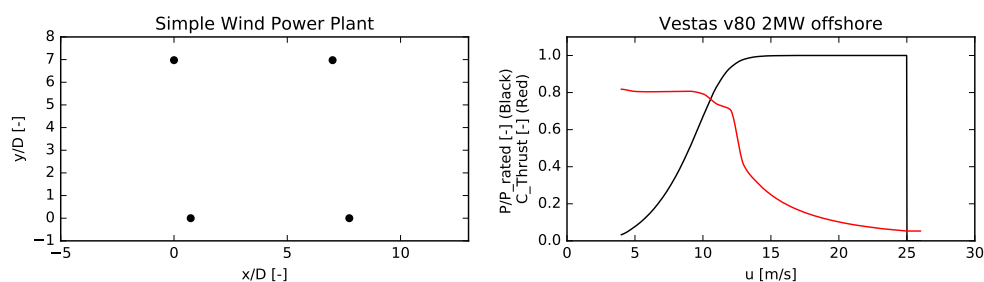


Figure 3. Simple wind power plant: (Left) layout and (Right) power and thrust coefficient curves for the turbines.

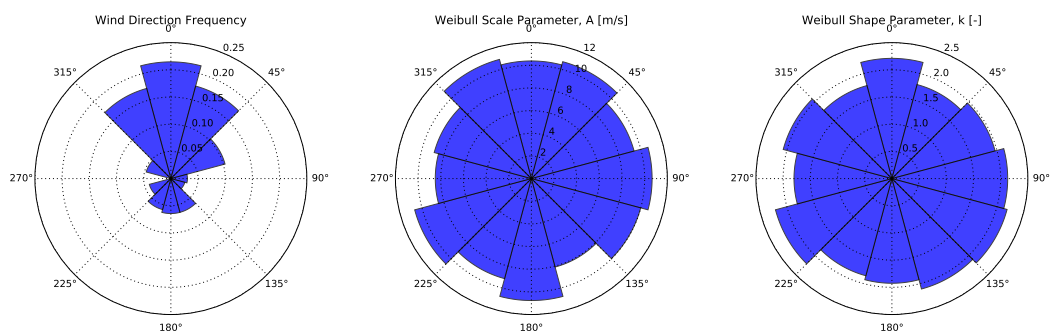


Figure 4. (Left) Wind rose of the site. (Middle) Weibull scale parameter for each sector. (Right) Weibull shape parameter for each sector.

The wind park's power curve, $P(u, \theta)$ is shown in figure 5 (Left). This power curve is computed using 30960 model evaluations: every 0.5 [deg.] and 0.5 [m/s]. This figure additionally shows an example of the resulting polynomial surrogate power curve. It can be observed that the power surrogate captures the rough behavior of power. As expected the polynomial surrogates present oscillations in the region of rated power as well as discontinuities between the regions; this does not cause any problem due to the fact that the integration in each region is done independently and since the oscillations around the *real* power curve cancel out when computing the $\mathbb{E}(P)$ or $\mathbb{V}(P)$. It might be of interest to visualize the polynomial surrogate because if the surrogate presents too many oscillations it means that the order of the polynomial is too high, and a further division of integration region can be executed.

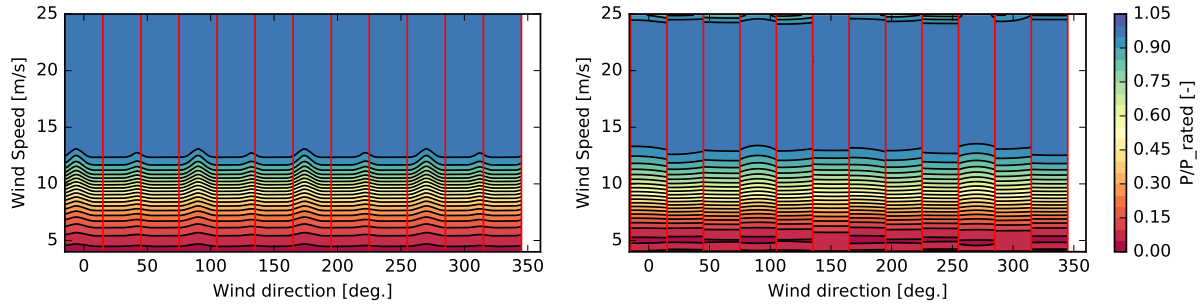


Figure 5. Power plant power curve: (Left) computed from $720 \times 43 = 30960$ model evaluations. (Right) Polynomial surrogate of the power curve order computed in $R_D = 12$ sectors, with $N_u = 8$ -th order in wind speed and $N_\theta = 5$ -th order in wind direction for a total of $12 \times 5 \times 9 = 540$ simulations.

The convergence of the expected power and of the standard deviation of power normalized by the rated wind farm power are shown in figure 6. The same numerical instabilities produced by the shifting of root locations can be observed. Furthermore it can be observed that the MEPC algorithm converges very fast to the desired values with relative error of order of 1%. This relative error is computed with respect the predictions with 30960 model simulations and it is show in the figure as the shaded purple region. It is important to remark that surrogates with a low polynomial degree, 2-nd in wind direction per sector and 4-th in wind speed, capture the expected power and its variance within 1% accuracy with around 10 times less model evaluations than the trapezoidal integration scheme used in the traditional binning method. Note that when $R_u = 2$, the regions are defined as: $[4, 14]$ and $[14, 25]$ [m/s], and the order on the rated power region is always $N_u[2] = 1$.

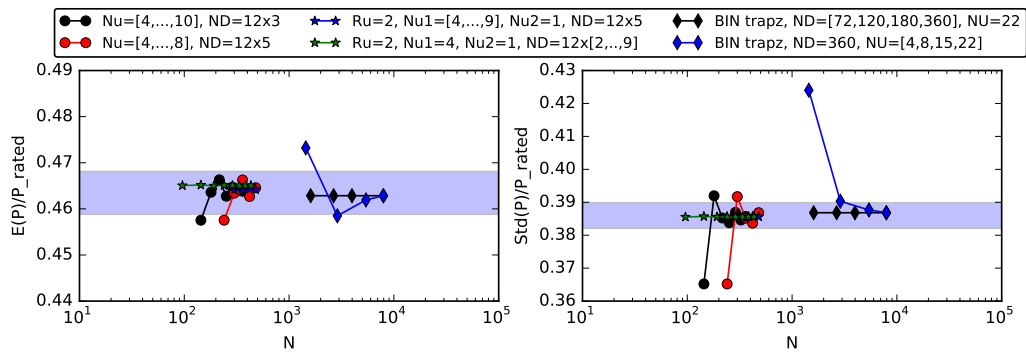


Figure 6. Convergence for different integration schemes vs. number of point evaluation N for the (left) normalized expected power $\mathbb{E}(P)$ and (right) normalized standard deviation of power, $\mathbb{S}(P)$ for number of model evaluations.

5 Real wind power plant case - Horns Rev 1

In this section the multi-element polynomial chaos technique is applied to the Danish offshore wind power plant Horns Rev 1 (HR1) co-owned by Vattenfall and DONG Energy. This wind farm is located in the Western coast of Denmark, at a distance of 9 to 12 [km] from the coast. This plant has been studied in several articles and it is one of the benchmark cases for wind

turbine wake models. The layout of HR1 is shown in figure 7, along with the power and thrust coefficient curves for the Vestas V80-offshore turbines. The wind rose and Weibull parameters for the site are shown in figure 8.

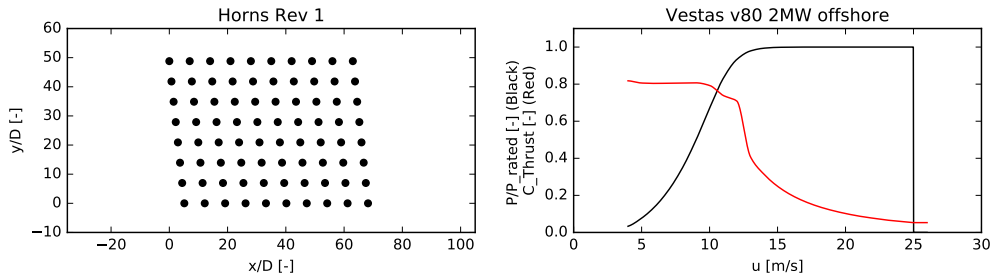


Figure 7. Horns Rev 1 power plant: (Left) layout and (Right) power and thrust coefficient curves for the turbines.

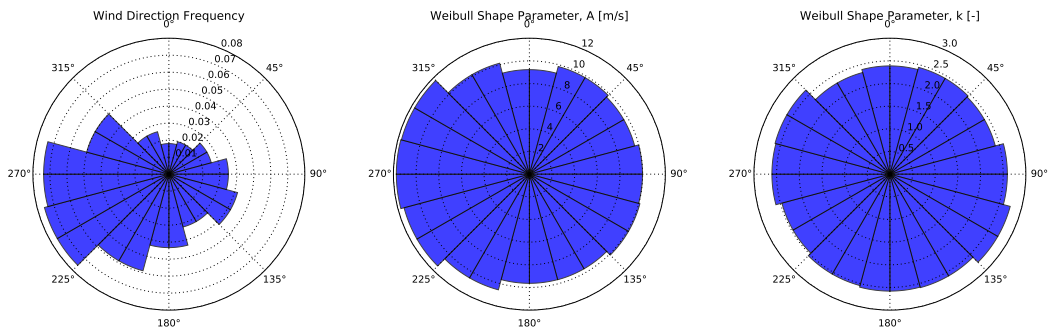


Figure 8. Horns Rev 1 plant: (Left) Wind rose of the site. (Middle) Weibull scale parameter for each sector. (Right) Weibull shape parameter for each sector.

The power curve of Horns Rev 1, $P(u, \theta)$ is shown in figure 9. As in the previous section the power curve is computed using 30960 simulations of G. C. Larsen's model [1]. An example of a polynomial surrogate for the power curve is also presented in this figure. As expected, the polynomial surrogate presents discontinuities between the sectors and it contains oscillations in the region of rated power. As discussed in the previous section, the discontinuities do not cause problems in the computation of AEP because each region is treated individually. Note that the 12 wind direction sectors were divided into 2 regions in order to reduce the degree of the polynomial basis required to capture the dependency on wind direction.

Convergence of power expectation and standard deviation can be observed in figure 10. It can be observed that the MEPC algorithm converges very fast to the desired values with relative error of order of 1%. This relative error is computed with respect the predictions done using 30960 model simulations and it is show in the figure as the shaded purple region. It is important to remark that surrogates with a low polynomial degree, 2-th in wind direction per sector and 4-th in wind speed, capture the expected power and its variance within 1% accuracy using 10 times less model evaluations than the trapezoidal integration scheme used in the traditional binning method. Note that when $R_u = 2$, the regions are defined as: $[4, 16]$ and $[16, 25]$ [m/s], and the order on the rated power region is always $N_u[2] = 1$.

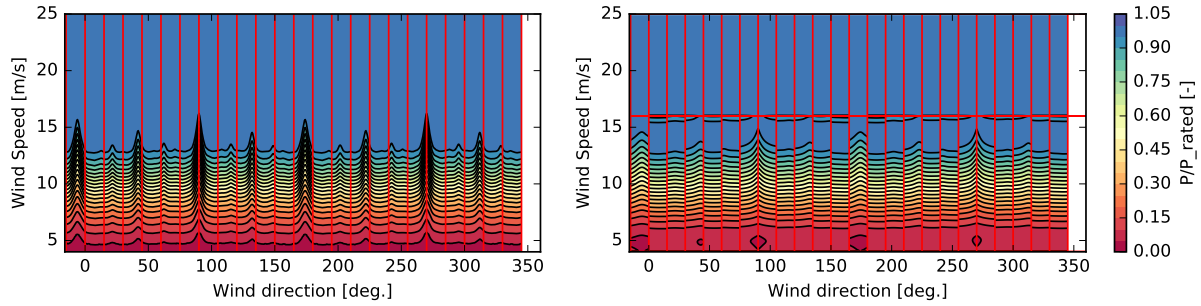


Figure 9. Horns Rev 1 power curve: (Left) computed from $720 \times 43 = 30960$ model evaluations. (Right) Polynomial surrogate of the power curve order computed in $R_D = 24$ sectors, with 4-th order in wind speed and 3-th order in wind direction for a total of $24 \times 3 \times 6 = 288$ simulations.

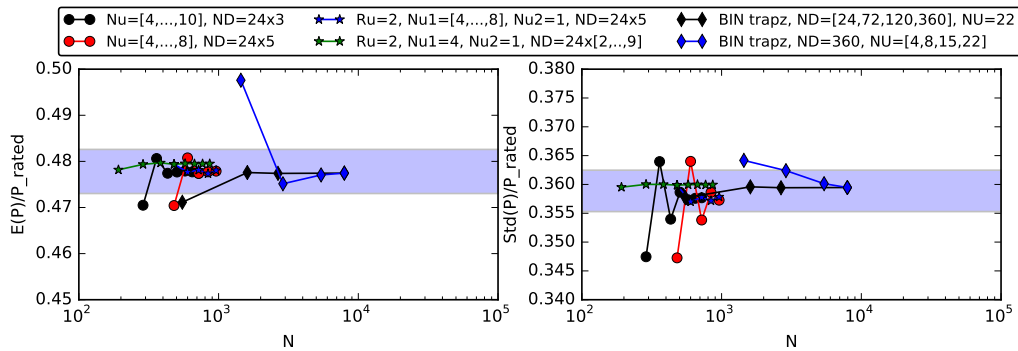


Figure 10. Convergence for different integration schemes vs. number of point evaluation N for the (left) normalized expected power $\mathbb{E}(P)$ and (right) normalized standard deviation of power, $\mathbb{S}(P)$ for number of model evaluations.

6 Discussion

The conceptual methodology of MEPC has been proven as having the potential to handle arbitrary local wind resources probability distributions. The methodology presented in the current paper has been implemented to handle user defined number/machine precision. Despite these efforts the polynomial basis building algorithm presents instabilities for large order of polynomial basis order. The quadrature rules are a fundamental step to find the statistical parameters of the power. Further research is still required to assure stability in the convergence rates of polynomial chaos for arbitrary Weibull distributions.

AEP production calculations with high fidelity flow models are restricted by their large computational requirements, in particular due to the large number of model simulations required to estimate the AEP with a certain level of accuracy. There is a need for methods to reduce the number of model evaluations required to compute power statistics. Furthermore analysis of uncertainty in power production have shown that the uncertainty in AEP can go up to 10% depending on the uncertainty in the input parameters [7]. The minimum requirement of AEP accuracy should be between 0.1 – 1%. MEPC method can achieve a relative error in the calculation of AEP smaller than 1% for a reduced number of evaluations. A reduction by a factor of 10 in comparison to current techniques can be achieved using MEPC with $192 = 24 \times 4 \times 2$ model simulations: this accounts for 2 regions of wind speed polynomial basis of orders $N_U = [4, 1]$ and a 2-nd order basis for wind direction.

The presented method is designed to be applied in optimization under uncertainty in which AEP calculations are going to be calculated on every optimization step (possible thousands of times). Each optimization step will propose a new wind power plant layout which makes the use of previous simulations impossible. Building the MEPC surrogate is a fast process, specially if the polynomial basis in wind direction and wind speed are precomputed and stored before the optimization technique. Furthermore, in optimization algorithms the accuracy of the AEP is gradually refined as the objective function increment diminishes, this can be achieved with the present method by increasing the order of the polynomial basis. Consequently a good estimation of AEP with few model simulations can speed up the process of optimization significantly and a more accurate AEP estimation will only be computed for the final steps.

Further investigation of the use of other types of functional basis are planed to be developed such as: (1) the use of polynomial basis for correlated wind velocity vector and turbulence intensity, (2) the use of wavelets for wind speed to deal with the discontinuities in the power curve and (3) a mixtures of polynomials and Fourier series for wind speed, wind direction and turbulence intensity. These techniques promise to have a better estimation of AEP but at the cost of increasing the number of model evaluations in comparison to the method presented in this article.

Acknowledgments

This work was supported by the International Collaborative Energy Technology R&D Program of the Korea Institute of Energy Technology Evaluation and Planning (KETEP), granted financial resource from the Ministry of Trade, Industry & Energy, Republic of Korea. (No. 20138520021140).

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